

Implications of quaternionic dark matter

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Abstract

Taking the complex nature of quantum mechanics which we observe today as a low energy effect of a broken quaternionic theory we explore the possibility that dark matter arises as a consequence of this underlying quaternionic structure to our universe. We introduce a low energy, effective, Lagrangian which incorporates the remnants of a local quaternionic algebra, investigate the stellar production of the resultant exotic bosons and explore the possible low energy consequences of our remnant extended Hilbert space.

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I. INTRODUCTION

Observational evidence and big bang nucleosynthesis bounds point to the existence of non-baryonic dark matter. The various dark matter candidates which have been explored in the literature all derive from attempts to extend the standard model either in purely phenomenological ways, or more fundamentally by seeking to incorporate the standard model into larger structures. Of the various fundamental approaches, one promising alternative which has not received much attention is a quaternionic quantum field theory (QQFT).

In fact, there are many ways to produce a quaternionic generalisation of the mathematical structure of quantum field theory. The axiomatic analysis of quaternionic quantum mechanics undertaken by Birkhoff and von Neumann [1] allows many formulations to be considered [2–7]. To constrain some of these possibilities we take the obvious success of the complex formulation of high energy physics as a clear sign that nature is *locally* describable by complex quantum field theory. By exploiting this local interpretation and the extended automorphism group of the quaternions we can introduce a local $SU(2)$ gauge symmetry which is naturally bound to our underlying construction. We do not propose to construct a full quantum field theory of quaternionic states. Rather, we consider the low energy effects of a quaternionic theory which, by some mechanism, has broken to the local complex structure we observe today. We wish to investigate if the resulting exotic bosons which arise as remnant degrees of freedom of the full $SU(2)$ theory can be identified as dark matter candidates. Such a notion would imply that quaternionic quantum effects may arise at galactic and cosmological scales.

The breaking is realised by singling out a particular quaternion which is ultimately identified with the imaginary unit of complex quantum field theory. In this way an exotic $U(1)$ survives to low energies. Our exotic bosons are coupled to normal matter via non-renormalisable terms in an effective Lagrangian. Our theoretical construct is sufficiently neat that only a relatively small number of parameters are needed to describe the new physics.

The crucial point is that we are seeking to encompass the introduction of new dark matter candidates and extensions of current physics from a fundamental shift in our mathematical description of nature. In this way we differ from simply introducing a phenomenological $SU(2)$, from which broader issues, such as the nature of the quantum theory, cannot be discussed. Indeed, this approach places new constraints on our effective Lagrangian which are not prescribed by a purely phenomenological theory.

Our aims in this paper are twofold: to explore if this notion is phenomenologically compatible with known astrophysical bounds and to introduce possible consequences of our underlying extended quantum mechanical structure. This will be addressed by first introducing our effective lagrangian in the next section, from which we calculate an astrophysical bound in the context of solar physics. Next, we consider the question of the cosmological significance of our exotic vector bosons. Finally, we shall speculate on the fundamental Lagrangian, the dynamics which underlies our effective theory and the possible phenomena which may be encapsulated by it.

II. MODEL

Studies of QQFT have most often started by considering a Hilbert space over the quaternionic field, for which the quantum mechanical amplitudes must, in general, be quaternionic and hence do not commute. We shall instead introduce a simplified model which, we maintain, retains the spirit of the original while holding out the best hope of connecting with experimental physics.

Our approach is to consider the complex nature we observe today as a low energy remnant of a full quaternionic theory at high energies. We thus first consider a space-time with a set of scalar fields $i(x_\mu)$, $j(x_\mu)$, and $k(x_\mu)$, which locally define a basis for the set of pure imaginary quaternions. Being local, invariance is required under the extended set of automorphisms of the quaternionic algebra which vary continuously over space-time, leading to an $SU(2)$ like exotic sector. We relate amplitudes at different places by considering local quaternionic

phase transformations of the type $\phi \rightarrow q\phi q^{-1}$, where q is a pure imaginary quaternion of unit magnitude. Note that at this stage all the fundamental fields are presumed to be quaternionic. It is worth noting, however, that we do not identify this sector with the fundamental SU(2) in electroweak theory, nor with any approximate SU(2) in hadronic physics.

The gauge-covariant derivative for this exotic gauge symmetry follows the usual pattern of Yang-Mills theory, but with a quaternionic valued potential, $D_\mu\phi = \partial_\mu\phi + \frac{1}{2}(Q_\mu\phi - \phi Q_\mu)$, where

$$Q_\mu(x) \equiv \bar{A}_\mu(x)i(x) + B_\mu(x)j(x) + B'_\mu(x)k(x). \quad (1)$$

Our choice of notation follows, *a posteriori*, from our interpretation of the phenomenological implications of our model.

From this we construct the field strength tensor, $K_{\mu\nu} = D_\mu Q_\nu - D_\nu Q_\mu$ and exhibit the Lagrangian density for the new gauge sector,

$$\mathcal{L}_Q = \frac{1}{4}K^{\mu\nu}K_{\mu\nu} + \frac{\lambda}{2}|D_\mu i|^2 + \frac{\lambda'}{2}|D_\mu j|^2. \quad (2)$$

This Lagrangian differs from that of Finkelstein, *et al.* [2] in that *two* pure imaginary quaternionic fields are singled out and considered as fundamental fields. This requires comment. Firstly, it is important to notice that the i and j fields are dimensionless. Scale is introduced by the constants λ, λ' , which are independent in the most general case, and have the dimension of mass squared (\mathcal{L}_Q is composed of dimension 4 operators). The components of the Q potential have the usual dimension of vector bosons. Furthermore, we have not included a $(\lambda''/2)|D_\mu k|^2$, as at each point of space-time, a consistent definition of the quaternionic algebra requires $k(x) \equiv i(x)j(x)$, so there are no additional dynamical degrees of freedom.

Quaternionic SU(2) transformations which leave the field $i(x)$ invariant define a U(1) subgroup of the general quaternionic group. We presume that via some mechanism this field has indeed been especially singled out. In this way our fully quaternionic theory can

be brought into contact with the usual complex quantum theory. In order to interpret the physical content of our model, we make a particular local quaternionic gauge transformation, to what we call the *i*-gauge (strongly analogous to the unitary gauge of electroweak physics). There always exists a quaternionic gauge transformation, q_i , taking $i(x_\mu)$ to a particular pure imaginary unit quaternion i , $q_i i(x_\mu) q_i^{-1} = i$ [2]. We associate this imaginary with the ordinary complex imaginary of complex quantum theory. In this particular gauge, the quaternionic potential is $Q_\mu(x) = \bar{A}_\mu(x) i + [B_\mu(x) + B'_\mu(x) i] j'(x)$, where $j'(x) = q_i j(x) q_i^{-1}$ is defined up to a complex phase,

$$j'(x) = \exp[\vartheta(x) i] j = \cos \vartheta(x) j + \sin \vartheta(x) k. \quad (3)$$

That is, we have introduced a fixed basis i , j and $k = ij$, and found that the remaining degrees of freedom reside in the complex phase of j' , and in the transformed vector potentials, \bar{A} , B and B' . The Lagrangian has become

$$\begin{aligned} \mathcal{L}_Q = & -\frac{1}{4}(\partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu + B_\mu B'_\nu - B_\nu B'_\mu)^2 + \{\text{permutations of } \bar{A}, B \text{ and } B'\} \\ & + \frac{\lambda}{2}(B_\mu^2 + B'^2_\mu) + \frac{\lambda'}{2}(\partial_\mu \vartheta + \bar{A}_\mu)^2 + \frac{\lambda'}{2}(B_\mu \sin \vartheta - B'_\mu \cos \vartheta)^2. \end{aligned} \quad (4)$$

Here, λ and λ' are *a priori* different, and we propose that $\lambda' \ll \lambda$. (In this regard, we might interpret the work of Finkelstein *et al.* [2] as being in the $\lambda' \rightarrow 0$ limit.) Then in a low to middle energy system, we might expect \bar{A} production to dominate. For the remainder of this letter, we shall only consider the case of $\lambda' \sim 0$. An investigation of the physical content of the $|D_\mu j|^2$ term is presented elsewhere [8] Finally, note that $D\phi$ reduces to $\partial\phi$ when ϕ is real.

We claim that the overwhelming success of conventional, complex quantum mechanics and quantum field theory is strongly indicative of the validity of a complex description of local physics (at least, on scales presently accessible to laboratory physics). Hence, taking the exotic quaternionic nature to be a high energy effect, we consider ordinary matter and gauge fields to be $i(x)$ complex. Our new U(1) sector then constitutes a “para-charged” region,

with our dark matter candidates B and B' being the only fields to exhibit a paracharge at low energies. However, since we expect that all fields will be fully quaternionic at some large energy scale we introduce an effective theory in which standard model physics is coupled to our broken quaternionic sector.

We follow the general discussion of [9], which explores possible interaction Lagrangian terms in the context of axionic physics. In our case, Q and K are pure imaginary quaternion, which leads us to propose an effective interaction Lagrangian which couples field densities to K^2 ,

$$\mathcal{L}_{\text{int}} = \frac{g_Q}{4\Lambda_Q^2} \phi^\dagger \phi K^2 + \frac{g_Q}{4\Lambda_Q^3} \bar{\psi} \psi K^2 + \frac{g_Q}{16\Lambda_Q^4} \text{tr} F^2 K^2. \quad (5)$$

where $\text{tr} F^2$ is the contribution from the usual $U(1)$ of electromagnetism. The exclusion of terms such as $\bar{\psi} \sigma \psi \cdot K$ is a constraint imposed by the formalism of our theory.

The scale Λ_Q is a characteristic quantity in QQFT, which we shall seek bounds on. Its role is equivalent to that of Λ_{QCD} in $\overline{\text{MS}}$ renormalisation schemes. Our theory is only effective. Consideration of the underlying fundamental theory is of paramount interest, but is expected to give equivalent results to the effective theory below the Λ_Q scale. It is natural to expect Λ_Q to be somewhere between the electroweak and Planck scales, but we do not rule out the possibility that this effective description may be inadequate for a description of top quark physics. This would place nontrivial predictions of our theory in an experimentally interesting region. These interaction terms are contact terms, describing multi-particle production above a mass pole (governed by λ') with Λ_Q^{-1} our perturbation theory parameter.

Our aim, now, is to find physical systems where the coupling of the sectors is sufficient to alter the predictions of standard physics. As our theory is not limited to extreme high energies, we shall next examine some astrophysical systems, of increasing scale, with the general idea of expecting to find new physics in systems with sufficient energy to produce excitations of the exotic fields, and at scales where the quaternionic curvature becomes noticeable.

III. APPLICATIONS

A. \bar{A} production in the Solar interior

We consider the possibility of a new luminosity channel for the Sun. Treating the solar interior as gas of moderately energetic electrons and photons, we calculate the production of our exotic vector bosons, using our effective interaction. We assume that this astrophysical system is sufficiently cool to lie well below the threshold for B , B' production, so that \bar{A} production dominates. A comparison with the solar (nuclear) energy production rate will allow us to place a bound on the scale of our effective theory, Λ_Q .

As our effective Lagrangian involves $N \geq 3$ body decays, we do not expect reabsorption to occur, and hence this may be an efficient cooling mechanism for a stellar body. On the other hand, the paraphotons, \bar{A} , can scatter off para-charged particles before escaping the stellar interior, which process will be suppressed by powers α_Q/Λ_Q^2 , and we shall not consider this effect in what follows. In fact, scattering of this kind provides the logical detection strategy; exploitation of the paraphotovoltaic effect.

The physical process we shall consider is photon conversion to \bar{A} “para-photons”. One might also consider Primakoff processes involving fermion loops coupling to the \bar{A} field; \bar{A} production by nucleons, which requires a model for the coupling of 1st generation quarks to the \bar{A} field within nuclear matter; paraphotonic bremsstrahlung by electrons in nuclear electrostatic fields; and by e^+e^- annihilation (though the last named effect is severely suppressed in the solar interior). In calculations of stellar production of axions [10] it is stated that compton processes contribute approximately 25% of the axion luminosity, with Primakoff processes dominating at low energies. We shall conservatively accept an uncertainty of a factor of 10 underestimation by ignoring these processes. In higher temperature stars, compton-type production dominates and our underestimation is expected to be reduced.

The matrix element for the process $e\gamma \rightarrow e\bar{A}\bar{A}$ is,

$$|\mathcal{M}_{\bar{A}\bar{A}}| = \left| \frac{eg_Q}{\Lambda_Q^3} \left(\bar{u}(p') \frac{1}{\not{p} + \not{k} - m} \not{\epsilon}_\gamma u(p) \right) \left(q^\mu \epsilon^\nu q'_\mu \epsilon'_\nu - q^\mu \epsilon^\nu q'_\nu \epsilon'_\mu \right) + \text{cross term} \right|, \quad (6)$$

where $\varepsilon, \varepsilon'$ label the polarisation vectors of the paraphotons (real paraphotons have transverse polarisations only).

We exploit the large differences between the Λ_Q , m_e and $\omega_\gamma \approx T_\odot$ scales to obtain the cross-section,

$$\sigma_{\bar{A}A} = \left(\frac{2\sqrt{2}}{7\pi} \right) \left(\frac{\omega_\gamma}{m_e} \right)^{5/2} \alpha \alpha_Q \frac{m_e^4}{\Lambda_Q^6}. \quad (7)$$

Calculation of the cooling rate achieved by this dark channel is by thermal averaging [10], with $E_{\bar{A}A} \approx \omega_\gamma$ representing 100% efficient photon conversion into paraphotons, $\dot{\epsilon}_{\bar{A}A}(T) = \frac{1}{\rho} \int dn_e dn_\gamma |v| E_{\bar{A}A} \sigma_{\bar{A}A}$, so

$$\dot{\epsilon}_{\bar{A}A}(T) = \left(\frac{2\sqrt{2}}{7\pi} \frac{\zeta(13/2)\Gamma(13/2)}{\pi^2} \right) \frac{n_e}{\rho} \left(\frac{T_\odot}{m_e} \right)^{13/2} \alpha \alpha_Q \frac{m_e^8}{\Lambda_Q^6}, \quad (8)$$

where we have used a thermal spectrum for $dn_\gamma(T)$, and $|v| = c$.

The nuclear energy production rate in the Sun is $\dot{\epsilon}_{\text{nuc}} \approx 2 \text{ ergs gm}^{-1} \text{ cm}^{-3}$. Assuming an effective temperature of $T_\odot = 10^7 \text{ K}$ and using a simple model for the Sun's chemical composition (70% Hydrogen, 30% Helium), we obtain the bound

$$1.81 \cdot 10^{20} \alpha_Q (m_e/\Lambda_Q)^6 < 2, \quad (9)$$

which we may convert to a lower bound on Λ_Q , assuming $\alpha_Q = \mathcal{O}(1)$: $\Lambda_Q > 2 \text{ GeV}$.

Note that Λ_Q scales with temperature as

$$(\Lambda_Q/\Lambda_Q^\odot)^6 = (\rho/\rho_\odot)(\dot{\epsilon}_{\text{nuc}}^\odot/\dot{\epsilon}_{\text{nuc}})(T/T_\odot)^{13/2}. \quad (10)$$

If we consider a red giant star, we have $T_{\text{RG}} \approx 10 - 100 T_\odot$, $\rho_{\text{RG}} \approx 100 \rho_\odot$, and $\dot{\epsilon}_{\text{nuc}}^{\text{RG}} \approx 10 \dot{\epsilon}_{\text{nuc}}^\odot$, and hence we have found a better bound: $\Lambda_Q^{\text{RG}} > 200 \text{ GeV}$.

We conclude that a Λ_Q of the order of the electro-weak scale, $\Lambda_{\text{EW}} = 250 \text{ GeV}$, is consistent with astrophysical bounds. Further, there is hope for signatures of new physics in the current generation of particle accelerator laboratories, in the form of missing energy not accounted for by neutrinos or neutral hadrons.

B. Cosmological population of exotic species

In the previous section, we have obtained a lower bound on the scale of our effective theory, which indicates that our theory is weakly coupled to the visible sectors. Consequently, any relic population of the \bar{A} , B , and/or B' bosons would have decoupled early in the thermal history of the universe. Consistency of our model with observational cosmology allows us to place bounds on the parameters occurring in the quaternionic free field Lagrangian Eq. (2).

Regarding a relic population of almost massless \bar{A} paraphotons, one can apply results from graviton physics [11]. The temperature of the relativistic species is $T_{\bar{A}} = (3.91/g_{*s})^{1/3}T_\gamma$, where $g_{*s} = \sum_b g_b(T_b/T_\gamma)^3 + (7/8)\sum_f g_f(T_f/T_\gamma)^3$ counts the degrees of freedom in all relativistic species at the decoupling temperature. When $T \approx \Lambda_q = 200 \text{ GeV}$, we have 4 quarks, 6 leptons and all the standard model bosons relativistic, so that $g_{*s} = 90$ and $T_{\bar{A}} \approx 1 \text{ K}$.

The contribution of a gas of \bar{A} radiation at this temperature is insufficient to over-close the universe. If we allow a small mass, which in our model corresponds to introducing $\lambda' > 0$, we have the bound $m_{\bar{A}} < (g_{*s}/g_{\bar{A}}) 12.8 \text{ eV} \approx 1 \text{ keV}$ (see [11]). The deeper implications of $\lambda' \neq 0$ are fascinating, as here we are restoring a dynamical j field, whose broader phenomenological implications include the possibility of cosmic strings [8].

Regarding the B and B' fields, we can say that a relic population of heavy bosons will contribute to the cold dark matter content of the universe. We expect broad agreement with heavy neutrino calculations [11]; the need to avoid over-closure translates to a substantial lower bound on λ in Eq.(2). A large difference in the λ and λ' scales may seem unnatural, and hence undesirable. This can be remedied by considering the case where all of \bar{A} , B and B' are light, hot species, which we will not look at in this paper.

IV. INTRODUCTION OF A TWO SCALE QUANTUM MECHANICS

In addition to the particle phenomenological effects of a broken $SU(2)$, from which para-photons and massive spin-1 particles are introduced, we will further exploit the extended complex structure of quaternions by exploring the possibility of a two scale quantum theory which is bound to the broken nature of our phenomenological construction. In this way we differ from previous quaternionic quantum mechanical approaches by demanding that, in our broken theory, the general quaternionic quantum theory is broken into two, orthogonal, complex quantum theories i.e. commensurate with the symmetry breaking $SU(2) \rightarrow U(1)$ the quantum mechanical Hilbert space *locally* breaks to two orthogonal parts. This is consistent with our effective construction where fields may be described as i complex or, as with B and B' , j , k complex. Denoting by \mathbf{C}_i the complex numbers arising about i and \mathbf{C}_H those about j and k , i.e. $jA_j + kA_k = k(A_k + iA_j)$, we can represent our generalised quantum mechanics via the set of numbers $(\mathbf{C}_i, k\mathbf{C}_i)$, where $k\mathbf{C}_i \cong \mathbf{C}_H$, which can be expressed as two orthogonal contributions via the anticommutator.

We note that, in our effective theory at least, each field is solely contained within one or the other quantum region. We therefore can express the functional dependencies as:

$$\begin{aligned} \mathcal{O} \in \mathbf{C}_i &\rightarrow \mathcal{O} = \mathcal{O}(q_1, p_1) \quad : \quad [q_1, p_1] = i\hbar_1 \\ \mathcal{O} \in \mathbf{C}_H &\rightarrow \mathcal{O} = \mathcal{O}(q_2, p_2) \quad : \quad [q_2, p_2] = i\hbar_2 \end{aligned} \tag{11}$$

Ideally, the orthogonality of the quantum regions can be employed to ensure that there is no interference between regions. Pursuing this, and recalling that i has been singled out by our construction, we can introduce a quantum bracket of the form $\{A_a, B_b\}_Q = -(1/2\hbar)\{[A_a, B_b], i\}i$, where $a, b = \mathbf{C}_i$ or \mathbf{C}_H indicating which region the operators in question correspond to. Such a bracket reduces to the usual commutation relation for $a = b = \mathbf{C}_i$ and provides a generalised quantum bracket for $a = b = \mathbf{C}_H$, where h is understood to correspond to \hbar_1 and \hbar_2 in each case respectively. Importantly, in the later case the result is an element of \mathbf{C}_i . In this way the ambiguity surrounding which complex number should

be employed in interpreting the quantum bracket is removed. For operators from different regions this quantum bracket vanishes.

An issue arises, however, when we recall that the quaternionic nature of operators is hidden in the Lagrangian (or Hamiltonian). Indeed, this implies that

$$d\mathcal{O}/dt = -(1/2\hbar)\{[\mathcal{O}, \mathcal{H}], i\}i \equiv 0 \quad \mathcal{O} \in \mathbf{C}_H, \quad (12)$$

so that under this quantum bracket the operators associated with \mathbf{C}_H are static. This provides a way to introduce a new background energy density but is however not a very interesting case. We are not, however, precluded from allowing our quaternionic fields to carry an explicit time dependence.

An alternative candidate quantum bracket which explicitly employs the quaternionic elements is not immediately obvious. We are compelled, then, to instead consider the implicit results of the two quantum regions which we encapsulate via the commutation relations: $[q_1, p_1] = i\hbar_1$ and $[q_2, p_2] = i\hbar_2$. Rather than represent $\mathcal{O} \in \mathbf{C}_H$ as $\mathcal{O} = j\mathcal{O}_j + k\mathcal{O}_k \in k\mathbf{C}_i$ we will separate out the hypercomplex part, k , so that $\mathcal{O} = \mathcal{O}_k + i\mathcal{O}_j = \mathcal{O}(q_2, p_2)$, and rely upon the usual quantum bracket $\{A_a, B_b\}_Q = -(1/2\hbar)[A_a, B_b]$, where the same conventions are observed as before. Note that this bracket will again vanish for operators from different regions by virtue of their dependence on different elements of position and momentum. Further, this bracket is again always an element of \mathbf{C}_i . It follows that

$$d\mathcal{O}/dt = -\frac{i}{\hbar_1}[\mathcal{O}, \mathcal{H}_1] - \frac{i}{\hbar_2}[\mathcal{O}, \mathcal{H}_2] - \frac{i}{\hbar_{av}}[\mathcal{O}, \mathcal{H}_I] - \frac{i\Delta\hbar^{-1}}{2}(J_2\mathcal{O}J_1 - J_1\mathcal{O}J_2) \quad (13)$$

where $\hbar_{av} = 2\hbar_1\hbar_2/\hbar_1 + \hbar_2$, $\Delta\hbar^{-1} = 1/\hbar_2 - 1/\hbar_1$, \mathcal{H}_1 refers to the free Hamiltonian in region 1, similarly for \mathcal{H}_2 and \mathcal{H}_I is the interaction Hamiltonian between the two quantum regions. J_1 and J_2 are the currents from regions one and two which comprise \mathcal{H}_I . It is known that the last term violates positivity but that unitarity can be restored via the introduction of noises in the currents. That is, the introduction of noises allows (13) to be an element of the class of Lindblad master equations, allowing embedding into an enlarged unitary dynamics [12].

We interpret this enlarged unitary dynamics as the extended dynamical set which exists beyond the low energy effective theory considered here. This is consistent with the notion that perturbations of the currents move to restore the full dynamics of the theory although this cannot be realised without explicitly enumerating the complete dynamics at high energies. Nevertheless, the need to embed within a larger dynamical set is consistent with our notion of an effective low energy theory. Interestingly, our low energy, quantum mechanically broken theory is no longer reversible.

Two interesting cases now present themselves. Taking $\hbar_2 \ll \hbar_1$ moves the new quantum mechanical sector to be realised as approximately classical in nature i.e. the quantum effects are suppressed. One possible application of such a regime is to the baryon asymmetry question. It is well known that quaternionic field theories are naturally CP violating. Further, with $\hbar_2 \ll \hbar_1$, it is to be expected that the decay lifetime of massive B and B' particles will be suppressed, allowing the interaction time to exceed the expansion time in the early stages of cosmological evolution. We can thus easily satisfy two of the conditions which lead to baryon asymmetry. Conversely, taking $\hbar_2 \gg \hbar_1$ will enhance the quantum field theoretic effects of our new sector. One immediate consequence would be to the creation of para-particle pairs in strong time varying gravitational fields. Indeed, an enhanced superluminescence may ensue, leading to rapid black hole evaporation rates.

Clearly, many interesting phenomena may be tackled in this way. The essential point, however, is that we can naturally introduce two different quantum regimes and that we are not compelled, *a priori*, to demand that $\hbar_2 \equiv \hbar_1$. Further, we would expect that such an approach would allow greater freedom in the interpretation of Λ_Q , perhaps allowing this scale to be significantly reduced while retaining phenomenological consistency.

V. SUMMARY AND CONCLUSION

By starting from a new fundamental basis, we have extended the standard particle physics description to a broken quaternionic theory whose consequences are realised at astrophys-

ical and cosmological scales. We have demonstrated that such an approach can naturally introduce new exotic species, satisfy phenomenological bounds and still contain sufficient additional structural freedom to incorporate additional interesting phenomena. It is not clear that a simple phenomenological introduction of an additional $SU(2)$ theory could accommodate such an interpretation, nor naturally support the new phenomenological constraints. It is intended that the possibilities surveyed here will be addressed in future work.

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